An Innovative Fast Powerful Method for Tackling Electromagnetic Eigenvalue Problems for Multistrip Transmission Lines

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***Abstract*—A full-wave electromagnetic technique is developed for the rapid and accurate calculation of dispersion characteristics in multiconductor and multilayer planar transmission lines. The proposed method is based on the Mrozowski and Przybyszewski algorithms. This powerful method calculates an approximate value of propagation constant at a desired frequency based on more accurate computations of the field distribution and propagation constant at a few selected frequency points. Comparison with previously accurate published data and numerical tests are first performed to confirm the accuracy of our procedure. Numerical results for several configurations are presented.**

***Index Terms*—Eigenvalue problems, full-wave analysis, planar transmission lines.**

I. Introduction

DETERMINING the normal modes of propagation of a guiding structure is known to be of fundamental importance. Due to the completeness of the set of normal modes [1], an arbitrary electromagnetic field inside the guiding structure can be expanded within this set. Moreover, the analysis of discontinuities between planar transmission lines, such as finline and microstrip configurations, has received increasing interest. Rigorous theoretical investigations of discontinuities by modal analysis have been reported by several authors (e.g., [2]–[4]) and a transverse resonance method based on an impedance matrix formulation was reported (e.g., [5]); determining high-order modes is not needed for this approach. The first approach method of analysis depends on the modal expansion concept. It is an application of the method of moments, in which both the basis and testing functions are the electromagnetic fields of the normal modes of propagation at both sides of the discontinuity. The main problem in this technique is the accurate determination of an approximately complete set of modes. Usually the weight of the mode in a field expansion series becomes smaller as the mode order goes higher so that determining of the first, say, N, modes is the actual need [6]. Therefore, one of the basic aims of computational electromagnetic is the calculation of dispersion characteristics of planar transmission lines. In fact, the design of passive and active components, which are of utmost importance in the realization of integral microwave circuits, wide-band characterization of any one of the configurations belonging to the generalized guiding structure, e.g., microstrip, finline, coplanar waveguide (CPW), etc, requires repetitive numerical solutions of a boundary value problem at successive frequency points. This is necessary for the calculation of both the fundamental and higher modes. Moreover, the study of higher modes is closely related to problems of discontinuity. This, in turn, requires an adequate selection of modes to obtain rapid convergence, avoiding overflow problems and minimizing CPU time. With a high computational cost for each solution, a lengthy overall time is needed to complete the computation for all points of interest. Several authors have recently suggested different approaches to overcome this problem [7]–[9]. In this paper, we have implemented a new technique based on the work recently developed by Przybyszewski *et al.* [9], which has been successful in dispersion analysis of waveguides. The method calculates an approximate value of propagation constant at a desired frequency based on more accurate computations of the field distribution and propagation constant at a few selected frequency points. The principal idea of the new algorithms proposed by Mrozowski [7] and Przybyszewski *et al.* [9] is to calculate a frequency independent optimal set of eigenfunctions, which satisfy all boundary conditions in large bandwidths. Although this stage of the process is performed only once, the choice and efficiency of the numerical technique used is of considerable importance as it may also be useful in the second stage of the process, depending on the type of method developed. The second stage involves expressing the fields at any frequency as a superposition of a small number of suitable eigenfunctions obtained in the first stage of the process. Finally, the method of moments can be used to find expansion coefficients. In the frequency domain, it involves solving a small system of linear equations. In this work, both the spectral-domain approach (SDA) [10] method and the singular value decomposition (SVD) [11] technique have been implemented to obtain an accurate set of modes. The simultaneous use of these two techniques (SDA and SVD) [12] offers the possibility of finding all the eigenmodes of any type of multiconductor and multilayer planar transmission lines. Likewise, these techniques are used to find the approximate values applying the new fast method. The proposed technique has been incorporated into the analysis of microstrip transmission lines and finline waveguides with great success.

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II. General Formulation

In order to carry out the fast efficient eigenmodes dispersion analysis for multilayer and multiconductor planar transmission

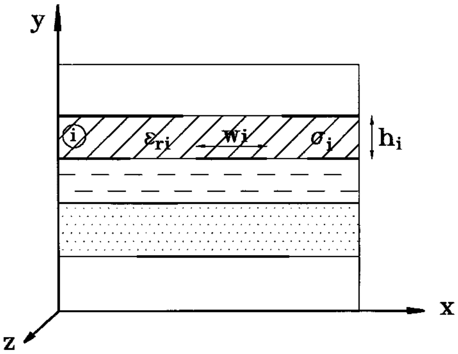

Fig. 1. Cross-sectional view of general multilayer and multiconductor transmission-line configuration.

lines, the hybrid fields are expressed by superposing TE-to-y and TM-to-y with Hertzian scalar potentials φ_i^e and φ_i^h, respectively, where the subscript i designates the regions. Fig. 1 shows the cross section of the general configuration.

The conductors are assumed to be infinitesimally thin and perfectly conducting, and the substrates are assumed to be low-loss ($\sigma_i = 0$) and nonmagnetic. By applying the continuity condition and the boundary condition equations, a dyadic Green's function can be derived. This function relates the surface current densities with the tangential electric fields [10]. To illustrate the application of the algorithms, we first need to be reminded of the most important equations developed by Przybyszewski *et al.* [9]. For this purpose, let us now consider Maxwell's equations in differential form and the constitutive relations. It states that

$$\nabla \wedge \left(\nabla \wedge \vec{E}\right) = -\frac{\delta \nabla \wedge \vec{B}}{\delta t} = -\mu \frac{\delta \nabla \wedge \vec{H}}{\delta t} \quad (1)$$

$$\nabla \cdot \left(\nabla \frac{\vec{D}}{\varepsilon\mu}\right) - \nabla^2 \cdot \frac{\vec{D}}{\varepsilon\mu} = \frac{\delta^2 \vec{D}}{\delta^2 t}. \quad (2)$$

If we assume that: 1) all components have the same propagation factor $e^{j(\omega t - \beta z)}$; 2) a source-free dielectric ($\nabla \cdot \vec{D} = 0$); and 3) ε and μ are not function of coordinates, by using the phasor representation, equation (2) with steady-state sinusoidal representation time dependence become

$$\frac{1}{\varepsilon\mu}\left(\frac{\delta^2 \vec{D}}{\delta x^2} + \frac{\delta^2 \vec{D}}{\delta y^2}\right) + \left(\omega^2 - \frac{\beta^2}{\varepsilon\mu}\right)\vec{D} = 0 \quad (3)$$

where β and ω are the propagation constant and the angular frequency, respectively.

The electric flux density $\left(\vec{D} = \varepsilon_o \varepsilon_{ri} \vec{E}\right)$ in each region satisfies the equation

$$L\vec{D} + \omega^2 \vec{D} - \beta^2 S\vec{D} = 0 \quad (4)$$

where L and S are operators defined as

$$L = \frac{1}{\varepsilon\mu}\frac{\delta^2}{\delta x^2} + \frac{1}{\varepsilon\mu}\frac{\delta^2}{\delta y^2} \qquad S = \frac{1}{\varepsilon\mu}.$$

In this way, once N discrete points have been obtained so that at each ω_i, β_i, and $\vec{D}_i$ as a function of ω_i, and β_i, $i = 1, \ldots, N$, each $\vec{D}_i$ satisfies the following equation:

$$L\vec{D}_i + \omega_i^2 \vec{D}_i - \beta_i^2 S\vec{D}_i = 0. \quad (5)$$

At other frequencies, to approach the solution using a superposition process

$$\vec{D}(\omega, \beta) = \sum_{i=1}^{N} c_i \vec{D}_i(\omega_i, \beta_i) \quad (6)$$

where c_i is a function of ω and β. Substituting (6) into (4) and simultaneously adding and subtracting $\sum c_i \omega^2 \vec{D}_i$, one gets

$$\sum_{i=1}^{N} c_i \left[L\vec{D}_i + \omega_i^2 \vec{D}_i + \left(\omega^2 - \omega_i^2\right)\vec{D}_i - \beta^2 S\vec{D}_i\right] = 0 \quad (7)$$

using (7), the equation to be solved becomes

$$\sum_{i=1}^{N} c_i \left[\left(\beta_i^2 - \beta^2\right) S\vec{D}_i + \left(\omega^2 - \omega_i^2\right)\vec{D}_i\right] = 0. \quad (8)$$

This equation is converted to a set of linear equations by taking the inner product of (8) with the eigenfunctions $\vec{B}_j^*$ as follows:

$$\left|M\left(\omega^2 I - \Omega^2\right) + SK^2\right| C - \beta^2 SC = 0 \quad (9)$$

with

$$\begin{aligned} \Omega &= \text{dia}\left[\omega_i^2\right] \\ K &= \text{dia}\left[\beta_i^2\right] \\ C &= \left[c_1, c_2, \ldots, c_n\right]^T \\ M_{k_i} &= \langle u_i, v_k^* \rangle = \int_s \left(\vec{D}_{ti} \times \vec{B}_{tk^*}^*\right) \cdot \vec{a}_z \cdot ds \\ S_{k_i} &= \langle S u_i, v_k^* \rangle = \int_s \left(\vec{E}_{ti} \times \vec{H}_{tk^*}^*\right) \cdot \vec{a}_z \cdot ds. \end{aligned} \quad (10)$$

Where S represents the cross section of multistrip and multilayer transmission lines and $\vec{a}_z$ is a unit vector in the z-direction.

The results are a system of homogeneous linear equations in terms of the unknown coefficients c_i. The above system problem leads to a homogeneous matrix equation $[A] \cdot X = 0$. The accuracy with which the previous equation can be solved is directly related to the accuracy with which the zeros of the $\det(A)$ can be detected.

Many numerical difficulties are encountered in the determination of the zeros of the characteristic equation because, as has already been shown, such lines can support complex modes. Hence, the zeros of the determinant of the characteristic matrix must be sought in the complex plane rather than on the real axis. Moreover, for certain combinations of structure parameters, a zero may be very close to a pole. Numerical techniques fail to find the zero in such cases. In the search for the complex solutions of the equation $[A] \cdot x = 0$, we have applied a general technique known as SVD [11], which is capable of solving homogeneous matrix equations without introducing solutions as poles. The above procedure is based on singular value composition, which is a powerful algorithm for dealing with matrices

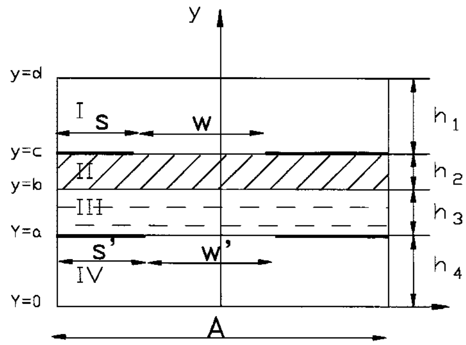

Fig. 2. Generalized finline.

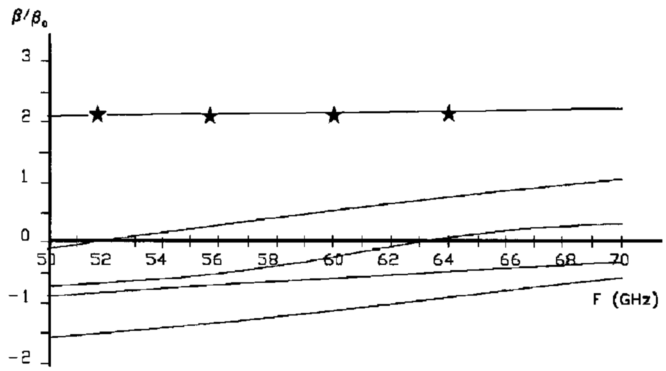

Fig. 3. Normalized propagation constants of the first five modes in a bilateral finline as a function of frequency. Parameters: $h1 = h4 = 1.0795$, $h2 = h3 = 0.1905$, $w = w' = 0.371$, $s = s' = 0.635$, $A = 1.27$. All the above dimensions are in millimeters. $\varepsilon_{r2} = \varepsilon_{r3} = 10$. The asterisk line is the approximate solution for the dominant mode obtained considering two basis functions: frequency (in gigahertz): 24.00, $\beta_1(3.9440261E-001, 0.00000E+000)$, frequency (in gigahertz) : 50.00, $(0.0000000E+000, -3.788118220)$.

that are either singular or else numerically very close to singular [13]. It has been demonstrated that by detecting the minima of the minimum singular value, instead of the zeros of the system determinant, the presence of poles can be eliminated, thus significantly simplifying the search algorithm and increasing the accuracy of the computations. Moreover, the detected value of the minimum itself provides a clear indication for the accuracy achieved.

The application of SVD is fundamental in both the approximate and accurate determination zeros because, by using this technique, we can eliminate the poles and steep gradients in the resolution of the characteristic equation rapidly and reliably. Solving the above system problem for $\beta^2(\omega)$, one gets the approximate dispersion characteristic constants as a function of the number and type of modes of the structure of interest. Once the characteristic equation is solved, the electromagnetic field can be determined.

III. Results and Conclusions

Even though the algorithm developed above is equally valid for any of the configurations belonging to multiconductor and multilayer planar transmission lines, we will only give results for the microstrip and finline configurations here because these structure are currently one of the most used and analyzed. Furthermore, these results can be considered as a first step

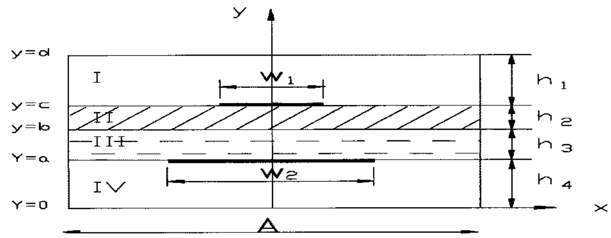

Fig. 4. Generalized microstrip.

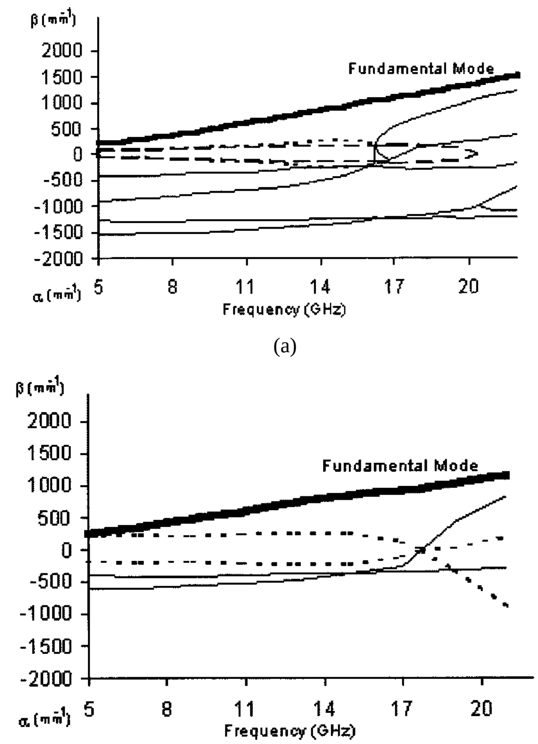

Fig. 5. Propagation constants versus frequency for suspended microstrip line.The dashed lines correspond to the real part of complex roots. (a) Accurate solution. Parameters: $h1 = h4 = 1.778$, $h2 = h3 = 0.5$, $w1 = 0.5$, $w2 = 0.0$, $A = 5$. All dimensions are in millimeters. $\varepsilon_{r2} = \varepsilon_{r3} = 20$. (b) Approximate results. The upper one is obtained considering the two first basis functions, while four basis functions has been used for the others curves: frequency (in gigahertz) 1.00 ($\beta_1 = 4.6682026E-002, -0.0000$), frequency (in gigahertz) 15.00 ($\beta_2 = 0.000E+000, -2.367E-001$), frequency (in gigahertz) 15.00 ($\beta_3 = 2.396E-001, -4.005E-001$), frequency (in gigahertz) 15.00 ($\beta_4 = -2.396E-001, -4.005E-001$).

toward the study of the other configurations, and is proof of the method's reliability.

The simultaneous use of these powerful techniques (SDA and SVD) in the root searching of homogeneous matrix equations can be considered as numerical procedures to determine the propagation constant of the dominant and higher order modes for accurate and approximate techniques. These results have been compared with accurate data, which has been obtained by applying the SDA method together with the SVD technique to obtain the set of modes that appears in finline and microstrip configurations. In SDA, fields and currents are first expanded in terms of some appropriate basis functions. Galerkin's method is then used to yield a homogeneous system of equations to determine the propagation constants. The repetition of all of these properties involved in the search for complex solutions is of

TABLE I
ACCURATE AND APPROXIMATE PROPAGATION CONSTANTS OF A SUSPENDED MICROSTRIP CONFIGURATION AS A FUCTION OF FREQUENCY. PARAMETERS AS FIGURE

Freq. (GHz)	β(mm^{-1})	Accurate	Approximate
1	β_1	4.668E-002	0.4670E-01
2	β_2	9.387E-002	0.9610E-01
3	β_3	1.421E-001	0.1450E+00
4	β_4	1.919E-001	0.1945E+00
5	β_5	0.2573E-00	0.2423E+00
6	β_6	0.2985E-00	0.2916E+00
7	β_7	0.3563E+00	0.3395E+00
8	β_8	0.4076E+00	0.3873E+00
9	β_9	0.4839E+00	0.4343E+00
10	β_{10}	0.5523E+00	0.4806E+00
11	β_{11}	0. 6254E+00	0.5256E+00
12	β_{12}	0.7018E+00	0.6389E+00
13	β_{13}	0.7613E+00	0.7389E+00
14	β_{14}	0.8636E+00	0.8783E+00
15	β_{15}	0.9479E+00	0.8672E+00
16	β_{16}	1.03382146	9.02365E-01
17	β_{17}	1.12042391	9.08953E-01
18	β_{18}	1.26143036	9.62428E-01
19	β_{19}	1.30088726	1.015901498
20	β_{20}	1.39183550	1.069476758

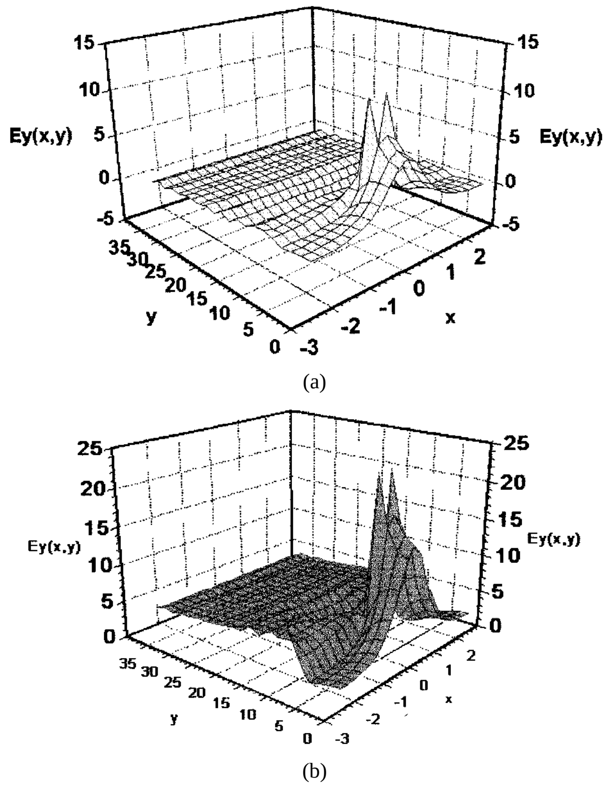

Fig. 6. Distribution of the $E_y(x, y)$ component for the first propagation mode versus x and y for: (a) accurate analysis corresponding with $\beta_1 = 0.840411$ at $\mathrm{freq.} = 15GHz$. (b) Approximate method corresponding to the approximate propagation constant $\beta_1 = 0.842699$ at 15 GHz; in this approximation, the set of basis functions used has been $\beta_1 = 0.053299$ at 1 GHz and $\beta_2 = 0.23336$ at 27.5 GHz, Parameters: $h1 = 3, h2 = h3 = 0.3175, h4 = 0.0, w1 = 0.56, w2 = 0.0, A = 5$. All the above dimensions are in millimeters. $\varepsilon_{r2}\varepsilon_{r2} = \varepsilon_{r3} = 10$.

utmost importance in the development of this algorithm and, therefore, precision in the search for the aforementioned solution will be fundamental in the suitability of the method.

The first step for checking the performance of the proposed approach is to verify that the approximation is correct for the structures where we know correct data. Firstly, the fundamental mode and higher modes have been found for a bilateral finline (Fig. 2). Fig. 3 shows the accurate propagation constants for the first five modes and the approximate first mode. The fundamental propagation constant mode is obtained with only two basis functions.

The second step has been to verify by comparison a second configuration, which is suspended microstrip. For suspended microstrip (see Fig. 4), the number of basis functions has been two for the fundamental mode and four for the higher modes in the frequencies ranging from 1 to 25 GHz. The propagation constants considered in Fig. 5(a) and (b) corresponds to accurate and approximate data, respectively. In Table I, it is possible to verify that the new algorithm gives very good results. Finally, it is possible to see the relevant correlation between the accurate and approximate field configuration (E_y) in Fig. 6(a) and (b), respectively, for a microstrip line. In order to obtain the above approximate configuration, we have used only two basis functions. Furthermore, to understand this good data it is necessary to consider the spectrum of modes at the cutoff frequency, which consists of the TEM term (only for a microstip line) and modes that are purely longitudinal section electric (LSE) (TE modes to y) or longitudinal section magnetic (LSM) (TM modes to y) terms. As the frequency increases, the interaction with the higher order eigenmodes becomes significant and a hybrid character becomes pronounced, which obviously is a result of the increasing contribution coming from the nearest mode. Taking a complex mode into consideration as a degeneration of TE and TM modes, this mode has enough information about the hybrid character of modes at high frequency. For this reason, the set of aforementioned basis solutions is sufficient for our purposes. In the light of the results obtained, it can be said that this new technique offers the following advantages: First, it is very simple to find the approximate situation of the complex roots, which is known to be one the most complex procedure involved in obtaining a complete set given the closeness of the poles and zeros [14], [15]. This advantage is not the aim of the method itself; however, through this study, we have verified for several configurations that with two basis functions, i.e., a two-by-two matrix, the position of that complex mode and its conjugate can be obtained within a very wide-frequency interval. The second advantage is to obtain fast and efficiently a complete set of modes in each structure. The above results forecast excellent possibilities for this algorithm if used in suitable functions.

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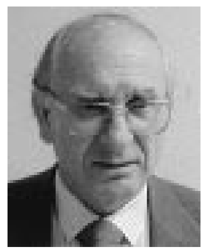

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